Practice Test
Transition to advanced mathematics
Spring 2016

1) Informally describe the following:

- Induction
- Relation
- Equivalence relation

Formally define the following:

- Induction
- Relation
- Reflexive
- Antisymmetric
- Total
- Transitive

2) Prove that for all $n \geq 1$:

$$\sum_{m=1}^{n} m = \frac{m(m+1)}{2}$$

3) Prove that for all $n \geq 1$:

$n^3 - n$ is divisible by 6

4) Prove that for all $n \geq 4$:

$$\prod_{m=1}^{n} \frac{1}{m} \leq \frac{1}{2^m}$$

5) Give two examples of relations on $\mathbb{R}$: one that is a function, and one that is not a function.

6) Give an example of a function. Be sure to describe it fully.

7) Formally define the relation that we know of as “mod 17”

8) Consider the relation below on $\mathbb{Z}$, that we'll call "Even mod 5".
xRy iff $x \equiv_5 y$ and x & y are either both even or both odd.

(a) Prove or disprove that "Even mod 5" is an equivalence relation.
(b) What is a simpler way to describe this relation?

9) Prove that 3^{-1} does not exist mod 6.

10) Find $2 + 6 \cdot 7$ mod 12.

11) Let R be an equivalence relation on a set S. Let $x, y \in S$. Prove that if $\bar{x} \cap \bar{y} \neq \emptyset$, then $\bar{x} = \bar{y}$

12) Let R be an equivalence relation on a set S. We proved that the equivalence classes of R partition S. In this problem we want to prove the part of that theorem that is below, so you can't use that theorem for this problem or else the logic would be circular:

Let $x, y \in S$. Prove that if $\bar{x} \cap \bar{y} \neq \emptyset$, then $\bar{x} \subseteq \bar{y}$

13) What does the LaTeX code below display as?

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$\Sum_{i=1}^{\infty}=\frac{\pi^2}{6}$
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